Droplets of mixtures of ^{3}He-^{4}He at finite vorticity

Hualin Shi [a], S. T. Chui [b,1]

[a] *Institute of Theoretical Physics, Chinese Academy of Sciences, Beijing, China*

[b] *Bartol Research Institute, University of Delaware, Newark, DE19716, USA*

Abstract

Stimulated by recent interests in mixtures of quantum fluids, we study the properties of ^{3}He-^{4}He mixture droplets using a phenomenological density functional. The structure of liquid drop with and without vortices is investigated using Monte Carlo simulations.

Key words: droplet, ^{3}He-^{4}He mixture, quantum fluid

There is a burgeoning interest in droplets of quantum fluids from different directions recently. These include droplets of Bose-Einstein condensed alkali atoms confined by magnetic and optical means[1], small droplets (1000 atoms) of He in a molecular beam[2], and large droplets (mm in diameter) levitated by a magnetic field[3]. While one component drops have received much interest, multicomponent systems are beginning to be looked at. In addition to their own intrinsic interest, droplets of mixtures provide a unique opportunity to test and enhance our understanding of finite size effects and the influence of boundaries on phase segregation effects.

Current Bose condensed alkali mixtures are mostly mixtures of Bose particles. ^{3}He-^{4}He are mixtures of Fermions and Bosons. In this work we present Monte Carlo simulations on the structure of ^{3}He-^{4}He mixture droplets with and without vortices as a function of their relative concentration. This follows our earlier work that focuses on the simpler aspects of this system[4]. At zero vorticity, because of the quantum pressure most of the ^{3}He stays on the outside with a small amount of ^{3}He remaining inside, as is previously reported.

The total free energy for a liquid-helium mixture of ^{3}He and ^{4}He can be expressed in a density functional form.[6] We approximate the volume integral of the energy functional by a discrete sum. We sample a lattice inside a sphere of radius $2R$ consisting of 40 sites along the diameter, making a total of 33398 sites. We choose the mesh size as 3 Å, which is smaller than the surface and interface thickness. We use the Metropolis algorithm, start our simulation with high temperature, then decrease it to a very low temperature step by step. This is an annealing process and the final configuration should be the lowest energy configuration. Because of the strong interactions between the helium atoms, the particle number is a very sensitive function of the chemical potential. We thus carry out the simulations with fixed particle numbers.

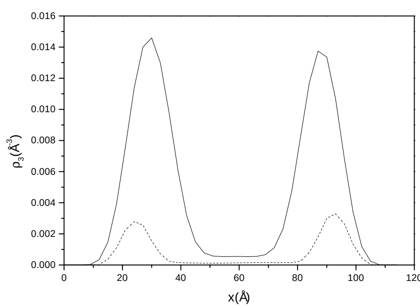

Fig. 1. Cross sections of density distributions of ^{3}He in the xy plane for z=0 with particle numbers $N_3 = 500$ and $N_4 = 3500$ (dashed line), $N_3 = 3000$ and $N_4 = 1000$ (solid line)

[1] Corresponding author. E-mail: chui@UDel.Edu

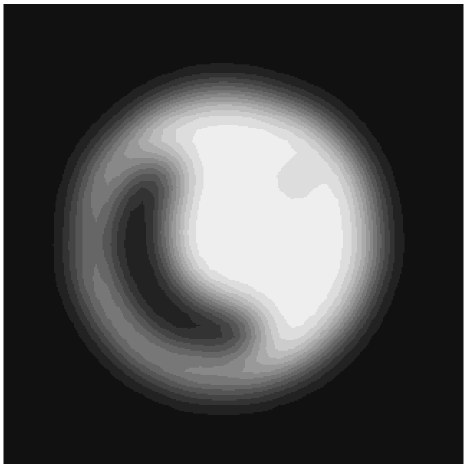

Fig. 2. Image of density distributions of ^{3}He with a vortex in ^{4}He. The vorticity is $m = 1$ and the particle numbers are: $N_3 = 3500$ and $N_4 = 500$.

Cross sections of the snap shots of the density profile for two different concentrations are shown in Fig. 1. The maximum bulk solubility of ^{3}He in the ^{4}He-rich phase is $\sim 6.6\%$ at zero temperature. When the ^{3}He fraction is small, there is a small amount ^{3}He inside the droplet, consistent with the bulk phase diagram. However, the concentration of ^{3}He is less than that in the bulk. We think this comes about because the ^{3}He inside must have the same chemical potential as that outside. Because of the free boundary condition the chemical potential of ^{3}He outside is lower than that in the bulk. When the ^{3}He number is very small, the solubility of ^{3}He into ^{4}He is much smaller than the bulk value. The effective attractive potential for ^{3}He atoms on the surface of ^{4}He provide a type of surface states, which are called Andreev states. ^{3}He atoms first accumulate on the free surface occupying Andreev states before start dissolving into the bulk. The recent change of the He3-4 mutual solubility in aerogels[5] is another example of the effect of boundary conditions on the phase segregation phenomena.

Taking the vortex along the z axis, the free energy functional of the vortex state is obtained with the addition of a centrifugal term $\hbar^2\rho_4\kappa^2/(2m_4r^2)$ associated with the superfluid flow where κ is the quantum circulation number and r is the distance to the z axis. At finite vorticity, ^{4}He moves outside because of the centrifugal force. Images of the snap shots of the density distributions at a low ^{4}He concentration ($N_3 = 3500$ and $N_4 = 500$) are shown in Fig. 2. The ^{4}He forms lumps and does seems to "wet" the ^{3}He completely. We emphasize that the pictures are **snap shots** and not **averages** of the denisty distribution. It is the density correlation between ^{3}He and ^{4}He that is being shown. Thus at low ^{4}He concentrations, the **density correlation** between ^{3}He and ^{4}He becomes asymmetric. Since ^{4}He is rotating, we expect the **average** density to be cylindrically symmetric. We think this phenomenon is related to the case of bulk ^{3}He-^{4}He mixture, where the votices lose their stability with increasing ^{3}He concentrations[**?**,**?**]. The more interesting question is how big will the damping be against the superfluid flow and how metastable is this configuration. We hope to investigate this in future publications.

This work is supported by a grant from NASA(NAG8-1427). We acknowledge helpful conversation with Ping Ao and V. Ryzhov.

References

[1] M.H. Anderson, J. R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, Science **269**, 198 (1995); C.C. Bradley et al., Phys. Rev. Lett. **75**, 1687 (1995); K.B. Davis et al., Phys. Rev. Lett. **75**, 3969 (1995); D. M. Stamper-Kurn et al., Phys. Rev. Lett. **80**, 2027 (1998).

[2] S. Grebenev, J. P. Toennies, and A. F. Vilesov, Science **279**, 2083 (1998).

[3] M. A. Weilert, D. L. Whitaker, H. J. Maris, and G. M. Seidel, J. Low Temp. Phys. **106**, 101 (1997).

[4] Hualin Shi and S. T. Chui

[5] M. Chan, N. Mulders, and J. Reppy, Physics Today, p30, August (1996).

[6] M. Guilleumas, D. M. Jezek, M. Pi, M. Barranco, and J. Navarro, Phys. Rev. B **51**, 1140 (1995).